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Spin Correlation in Muon-Pair Production.

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Summary. — The correlation of the muon spins in muon pair production by unpolarized photons is calculated in Born approximation. The longitudinal correlation coefficient is found to be sensitive to a cut-off in the muon propagator at 0.1 fermi.

1. – Introduction.

The correlation of the electron spins in electron-pair production by photons was studied by Olsen and Maximon (1). These authors were interested in the high-energy case and used Sommerfeld-Maue type wave functions in order to take account of screening and Coulomb effects. Since in the high-energy case the electrons and positrons are produced in a very narrow forward cone, Olsen and Maximon integrated over the electron and positron directions and obtained the spin correlation of the whole electron and positron «beams».

In the following we shall consider the spin correlation of the muon pairs in muon-pair production by unpolarized photons. We are interested in energies not too far above threshold, for which we have an extended angular distribution rather than forward production. In this case the momentum transfer to the nucleus is large so that screening becomes unimportant. Disregarding the problem of Coulomb and radiative corrections, we calculate the differential

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(1) H. Olsen and L. C. Maximon: *Phys. Rev.*, **114**, 887 (1959), referred to as OM.

cross-section for definite spin directions in the Born approximation. From this we get the differential spin correlation coefficient. Introducing a cut-off in the muon propagator we study the influence of an eventual breakdown of muon quantum electrodynamics on the spin correlation coefficient.

2. – The cross-section in Born approximation.

The differential cross-section for pair production with definite spin orientation is (2):

$$\mathrm{d}\sigma = \frac{e^4}{8(2\pi)^5}\,\frac{|\boldsymbol{p}|\,|\boldsymbol{q}|\,\mathrm{d}q_0}{k_0^3}\,\mathrm{d}\Omega_p\,\mathrm{d}\Omega_q\,T_{\mu\nu}A_\mu(\boldsymbol{k}-\boldsymbol{p}-\boldsymbol{q})A_\nu(\boldsymbol{p}+\boldsymbol{q}-\boldsymbol{k})\,, \tag{1}$$

with

$$\left\{\begin{aligned} T_{\mu\nu} = \frac{k_0^2}{2}\,\mathrm{Spur}\Bigg\{&\left[\gamma_\mu\,\frac{i\gamma(q-k)+m}{2qk}\,\gamma_\lambda+\gamma_\lambda\,\frac{i\gamma(p-k)-m}{-2pk}\,\gamma_\mu\right]\cdot\\ &\cdot\frac{1}{2}(1-i\gamma_5\gamma t)(i\gamma q+m)\cdot\left[\gamma_\lambda\,\frac{i\gamma(q-k)+m}{2qk}\,\gamma_\nu+\gamma_\nu\,\frac{i\gamma(p-k)-m}{-2pk}\,\gamma_\lambda\right]\cdot\\ &\cdot\frac{1}{2}(1+i\gamma_5\gamma s)(i\gamma p-m)\Bigg\}\,, \end{aligned}\right. \tag{2}$$

where k, p, q are the momenta of the incident photon and of the two muons, respectively, m is the muon mass. s and t are the usual covariant spin operators for the muons (3):

$$\left\{\begin{aligned} s &= \left(\boldsymbol{\eta}+\frac{\boldsymbol{\eta}\cdot\boldsymbol{p}}{m(p_0+m)}\,\boldsymbol{p}\,,\ i\,\frac{\boldsymbol{\eta}\cdot\boldsymbol{p}}{m}\right),\\ t &= \left(\boldsymbol{\zeta}+\frac{\boldsymbol{\zeta}\cdot\boldsymbol{q}}{m(q_0+m)}\,\boldsymbol{q}\,,\ i\,\frac{\boldsymbol{\zeta}\cdot\boldsymbol{q}}{m}\right), \end{aligned}\right. \tag{3}$$

where $\boldsymbol{\eta}$ and $\boldsymbol{\zeta}$ are the spatial polarization vectors. Carrying out the trace operations in (2), we get

$$T_{\mu\nu} = \frac{k_0^2}{2}\left[\frac{A_{\mu\nu}}{4(qk)^2}+\frac{B_{\mu\nu}}{4(pk)^2}-\frac{C_{\mu\nu}}{4(qk)(pk)}\right], \tag{4}$$

(2) We use hermitian γ-matrices and the metric $x^2=\boldsymbol{x}^2+x_4^2$.

(3) H. A. Tolhoek: *Rev. Mod. Phys.*, **28**, 277 (1956).

with

$$\text{(5a)}\quad \begin{cases} A_{\mu\nu} = 8m^2 \left\{\frac{1}{2}\,\delta_{\mu\nu}\left[(1-st)(pq-pk-qk-m^2)+\frac{1}{m^2}\,(pk)(qk)+\right.\right. \\ \quad \left.+(sq-sk)(tp-tk)\right]-(1-st)p_\mu q_\nu+(sk-sq)p_\mu t_\nu+(tp-tk)(s_\mu k_\nu-s_\mu q_\nu)+ \\ \quad \left.+(pq-pk-qk-m^2)s_\mu t_\nu+\left(1-st-\frac{qk}{m^2}\right)k_\mu p_\nu\right\}, \end{cases}$$

$$\text{(5b)}\quad \begin{cases} B_{\mu\nu} = 8m^2 \left\{\frac{1}{2}\,\delta_{\mu\nu}\left[(1-st)(pq-pk-qk-m^2)+\frac{1}{m^2}\,(pk)(qk)+\right.\right. \\ \quad \left.+(sq-sk)(tp-tk)\right]-(1-st)p_\mu q_\nu+(tk-tp)q_\mu s_\nu+(sq-sk)(t_\mu k_\nu-t_\mu p_\nu)+ \\ \quad \left.+(pq-pk-qk-m^2)s_\mu t_\nu+\left(1-st-\frac{pk}{m^2}\right)k_\mu q_\nu\right\}, \end{cases}$$

$$\text{(5c)}\quad \begin{cases} C_{\mu\nu} = 8\,\{k_\mu k_\nu[m^2-(st)(pq)+(sq)(tp)]+ \\ \quad + k_\mu p_\nu[(kq)(st)-(pq)(1-st)-(sq)(kt+pt)]+ \\ \quad + k_\mu q_\nu[(kp)(st)-(pq)(1-st)-(tp)(ks+qs)]+ \\ \quad + k_\mu s_\nu[(kt)(pq)-(pt)(kq+m^2)]+k_\mu t_\nu[(ks)(pq)-(qs)(kp+m^2)]+ \\ \quad + p_\mu p_\nu[(kq)(1-st)+(kt)(qs)]+ \\ \quad + p_\mu q_\nu[(2pq-pk-qk)(1-st)+(kt)(qs)+(ks)(pt)]+ \\ \quad + p_\mu s_\nu[(kq)(pt)-(kt)(pq-m^2)]+ \\ \quad + p_\mu t_\nu[(qs)(2pq-kp-2kq)-(ks)(pq+m^2)]+ \\ \quad + q_\mu q_\nu[(kp)(1-st)+(ks)(pt)]+ \\ \quad + q_\mu s_\nu[(pt)(2pq-2kp-kq)-(kt)(pq+m^2)]+ \\ \quad + q_\mu t_\nu[(kp)(qs)-(ks)(pq-m^2)]+ \\ \quad + s_\mu t_\nu(pq)2(kp+kq-pq+m^2)+ \\ \quad + \delta_{\mu\nu}[(1-st)(pq)(kp+kq-pq+m^2)-(st)(kp)(kq)+ \\ \quad + (kt)(sq)(kp+m^2)+(ks)(pt)(kq+m^2)- \\ \quad - (pq)\big((ks)(kt)+(pt)(qs)\big)+(pt)(qs)\big((kp)+(kq)\big)]\}. \end{cases}$$

These expressions for $A_{\mu\nu}$, $B_{\mu\nu}$, and $C_{\mu\nu}$ are to be symmetrized in μ and ν, but for the following, the form given is sufficient.

Inserting for $A_\mu(\boldsymbol{k}-\boldsymbol{p}-\boldsymbol{q})$ the Coulomb potential, we have for (1):

$$\text{(6)}\qquad \mathrm{d}\sigma = \left(\frac{Z}{2\pi}\right)^2\left(\frac{e^2}{4\pi}\right)^3 \frac{|\boldsymbol{p}|\,|\boldsymbol{q}|\,\mathrm{d}q_0}{k_0^3}\,\mathrm{d}\Omega_p\,\mathrm{d}\Omega_q\left[-\frac{T_{44}}{(\boldsymbol{k}-\boldsymbol{p}-\boldsymbol{q})^4}\right].$$

3. – The spin correlation of the muon-pair.

In contrast to the case of high-energy electron-pair production treated by OM, in which essentially forward production is observed, in our case the muons are supposed to be detected by small counters placed under some angle to the photon beam. The spin of the positive muon shall be measured in the direction $\boldsymbol{\eta}$, the spin of the negative one in the direction $\boldsymbol{\zeta}$. Then we define the spin correlation coefficient C by

$$C = \frac{\mathrm{d}\sigma(\boldsymbol{\zeta}, \boldsymbol{\eta}) - \mathrm{d}\sigma(\boldsymbol{\zeta}, -\boldsymbol{\eta})}{\mathrm{d}\sigma(\boldsymbol{\zeta}, \boldsymbol{\eta}) + \mathrm{d}\sigma(\boldsymbol{\zeta}, -\boldsymbol{\eta})}. \tag{7}$$

Inserting the cross-section (6), (4) into (7), we get:

$$C = \frac{A^{\zeta}_{44}\dfrac{pk}{qk} + B^{\zeta}_{44}\dfrac{qk}{pk} - C^{\zeta}_{44}}{A^{0}_{44}\dfrac{pk}{qk} + B^{0}_{44}\dfrac{qk}{pk} - C^{0}_{44}}. \tag{8}$$

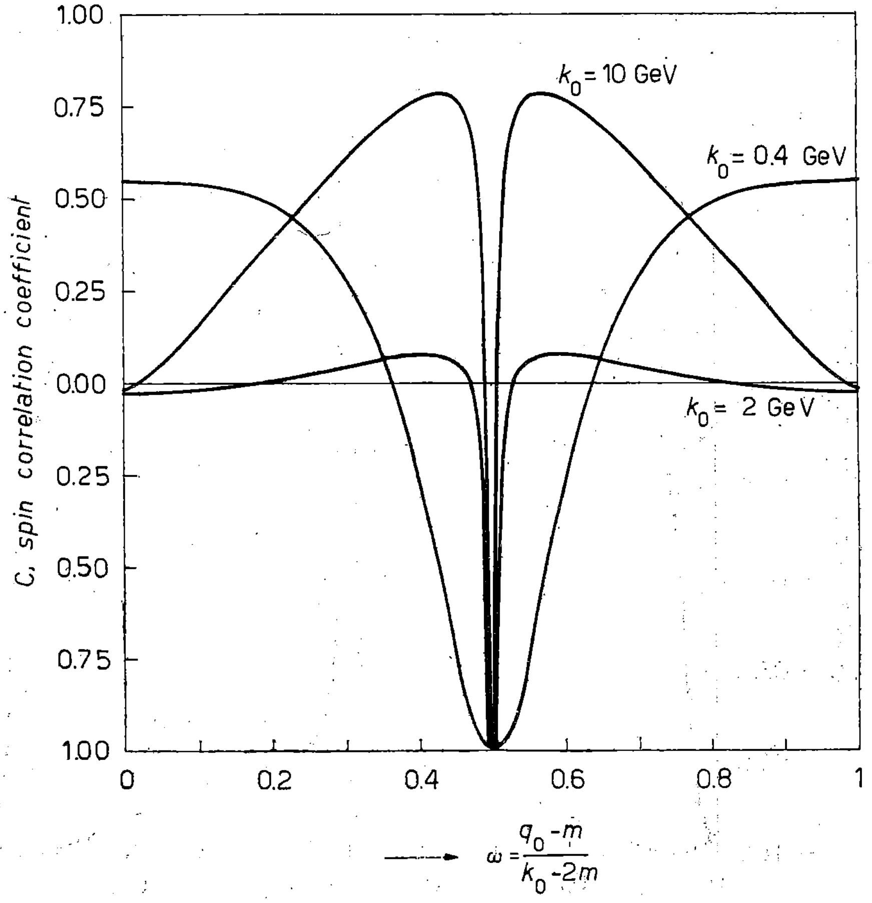

Fig. 1. – Spin correlation coefficient C as a function of $\omega = (q_0 - m)/(k_0 - 2m)$, the fraction of the kinetic energy of one muon to the total kinetic energy. Coplanar case $\boldsymbol{k}\cdot(\boldsymbol{p}\times\boldsymbol{q}) = 0$, $\varphi_1 = \varphi_2 = 10°$, spins transverse in the $\boldsymbol{p}, \boldsymbol{q}$ plane.

Here A_{44}^{ζ} is the part of A_{44} containing $\boldsymbol{\zeta}$ and $\boldsymbol{\eta}$, while A_{44}^{0} is the polarization-independent part of A_{44}, analogously B_{44}^{0} etc.

As in the Born approximation the screening correction factors out in the differential cross-section, screening does not influence C. But screening is unimportant anyway because of the large momentum transfers.

In the following we consider the special case that the two produced muons and the incident photon are coplanar, *i.e.* $\boldsymbol{k}\cdot(\boldsymbol{p}\times\boldsymbol{q})=0$. We evaluated (8) numerically for the longitudinal spin case and for two transverse spin directions. The two muons are always chosen to be produced symmetrically with respect to the photon, *i.e.* $\varphi_1=\varphi_2$, where φ_1 is the angle between $\boldsymbol{k}$ and $\boldsymbol{p}$, φ_2 the angle between $\boldsymbol{k}$ and $\boldsymbol{q}$.

Having chosen the muons and the photon coplanar, we can measure the transverse polarization of the muons *in* the muon-photon plane or *orthogonal* to this plane. It turns out, however, that these two cases give qualitatively the same results for the energies and angles considered. Therefore we restrict ourselves to the case of the spins orthogonal to the muon-photon plane. The transverse spin correlation is plotted in Fig. 1 for different photon energies as a function of the kinetic energy of one of the muons. The corresponding results for the longitudinal-spin case ($\boldsymbol{\zeta}\|\boldsymbol{q}$, $\boldsymbol{\eta}\|\boldsymbol{p}$) in the coplanar situation (with)

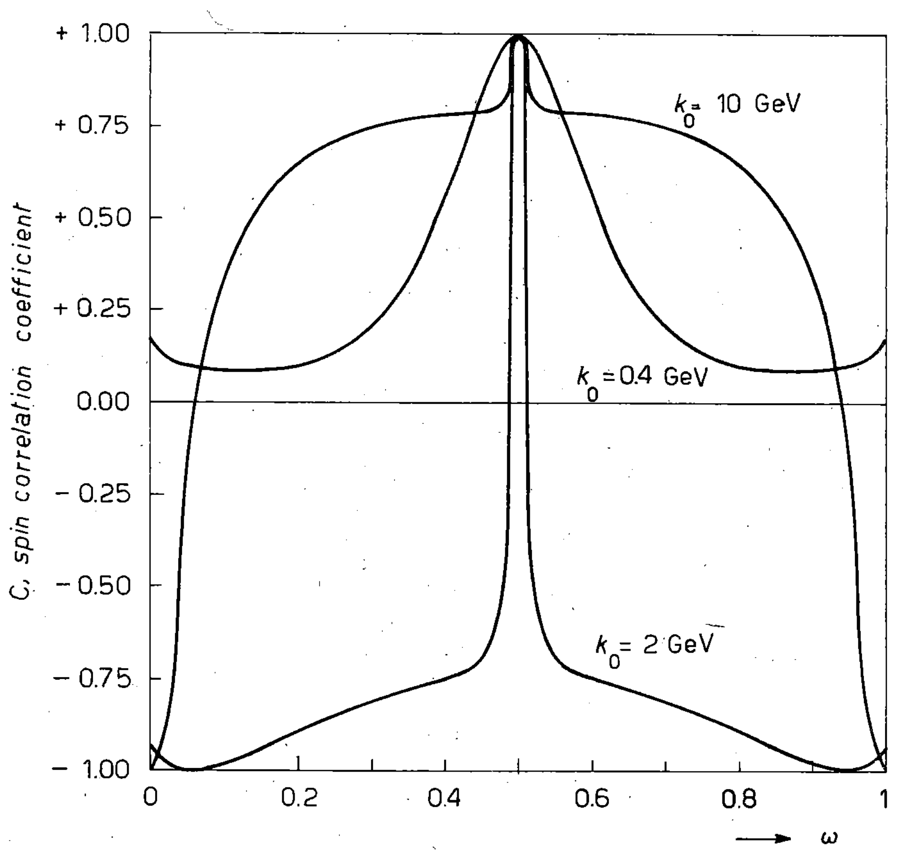

Fig. 2. – Spin correlation coefficient C for the coplanar case and $\varphi_1=\varphi_2=10°$, as in Fig. 1, but spins longitudinal.

$\varphi_1 = \varphi_2 = 10°$ are given in Fig. 2. The dependence on φ_1, φ_2 is illustrated by the comparison of Fig. 2 with the results for $\varphi_1 = \varphi_2 = 30°$ given in Fig. 3.

The symmetry of the curves with respect to an exchange of the two muons is of course due to the use of the Born approximation. Whereas in OM the correlation coefficient integrated over all angles φ_1 and φ_2 was found to be

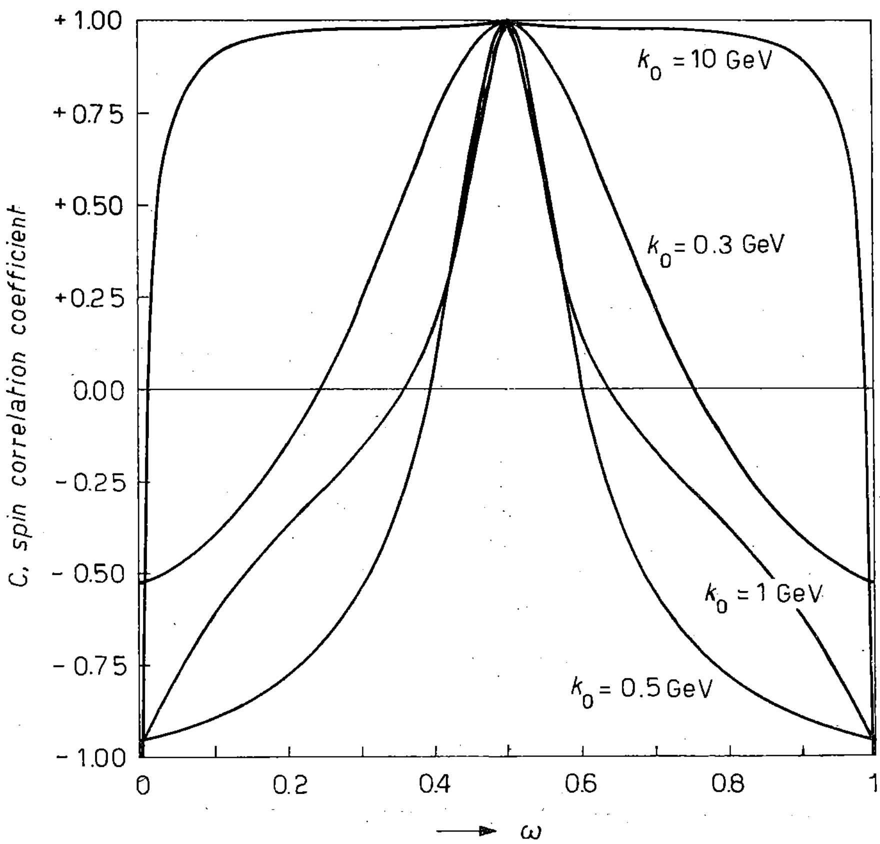

Fig. 3. – Spin correlation coefficient C for the coplanar case and longitudinal spins, as in Fig. 2, but $\varphi_1 = \varphi_2 = 30°$.

almost independent of the photon energy, this is not the case for our differential correlation coefficients. The correlation becomes 100% in the case of $\varphi_1 = \varphi_2$, $\boldsymbol{k} \cdot (\boldsymbol{p} \times \boldsymbol{q}) = 0$, and equal energy of the muons, as may also be verified by direct calculation from eq. (8). The integrated correlation coefficient has not this peak reaching 100% because it receives also contributions with $\varphi_1 \neq \varphi_2$.

4. – Influence of a cut-off in the muon propagator.

In the last paragraphs we studied the muon spin-correlation assuming the validity of quantum electrodynamics for the muon interactions. A possible breakdown of muon quantum electrodynamics may be described by a modi-

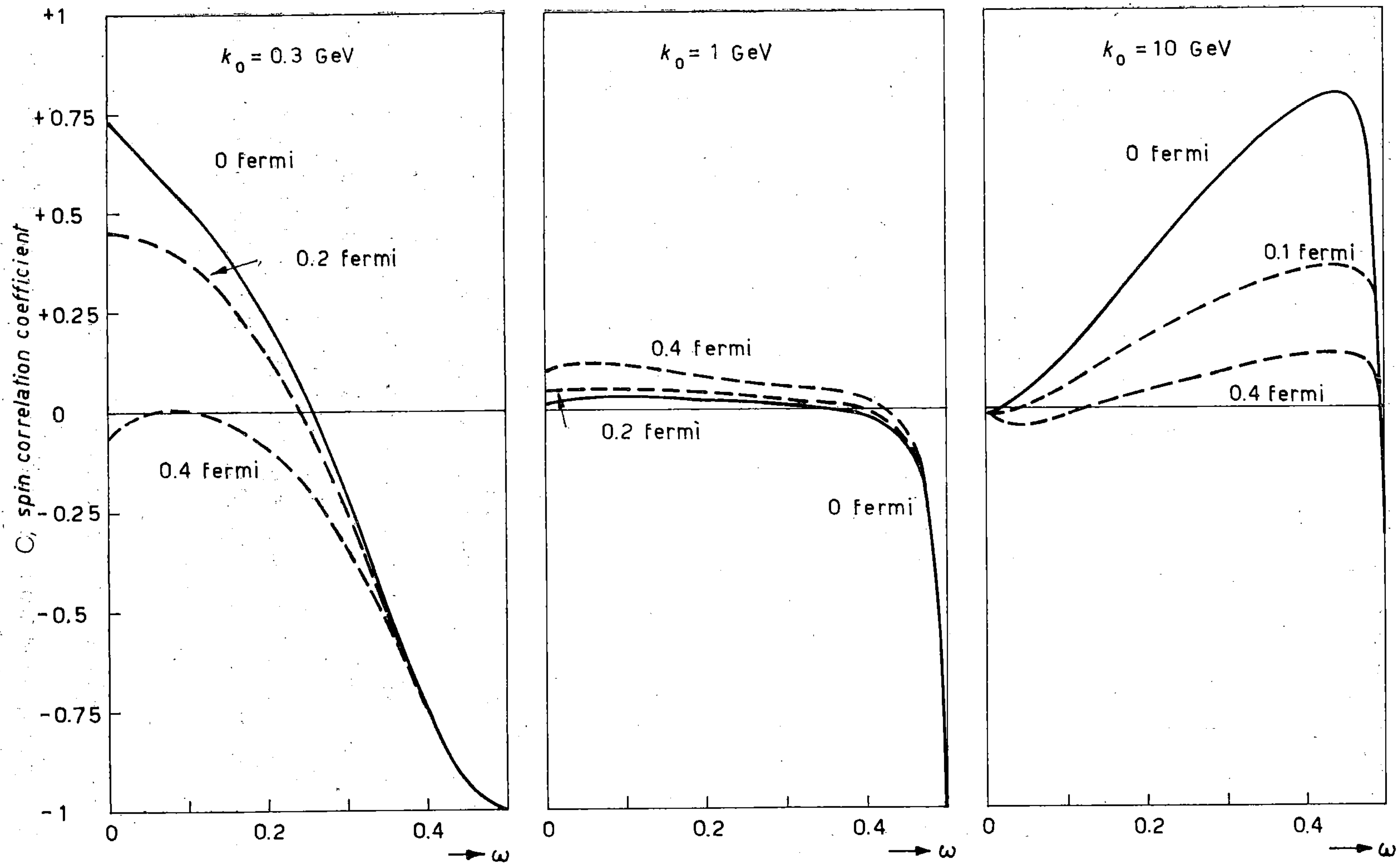

Fig. 4. – Spin correlation coefficient C for the coplanar transverse case with $\varphi_1=\varphi_2=10^\circ$, as in Fig. 1 but with a cut-off Λ in the muon propagator ($\Lambda^{-1}=0$ fermi is the case without cut-off).

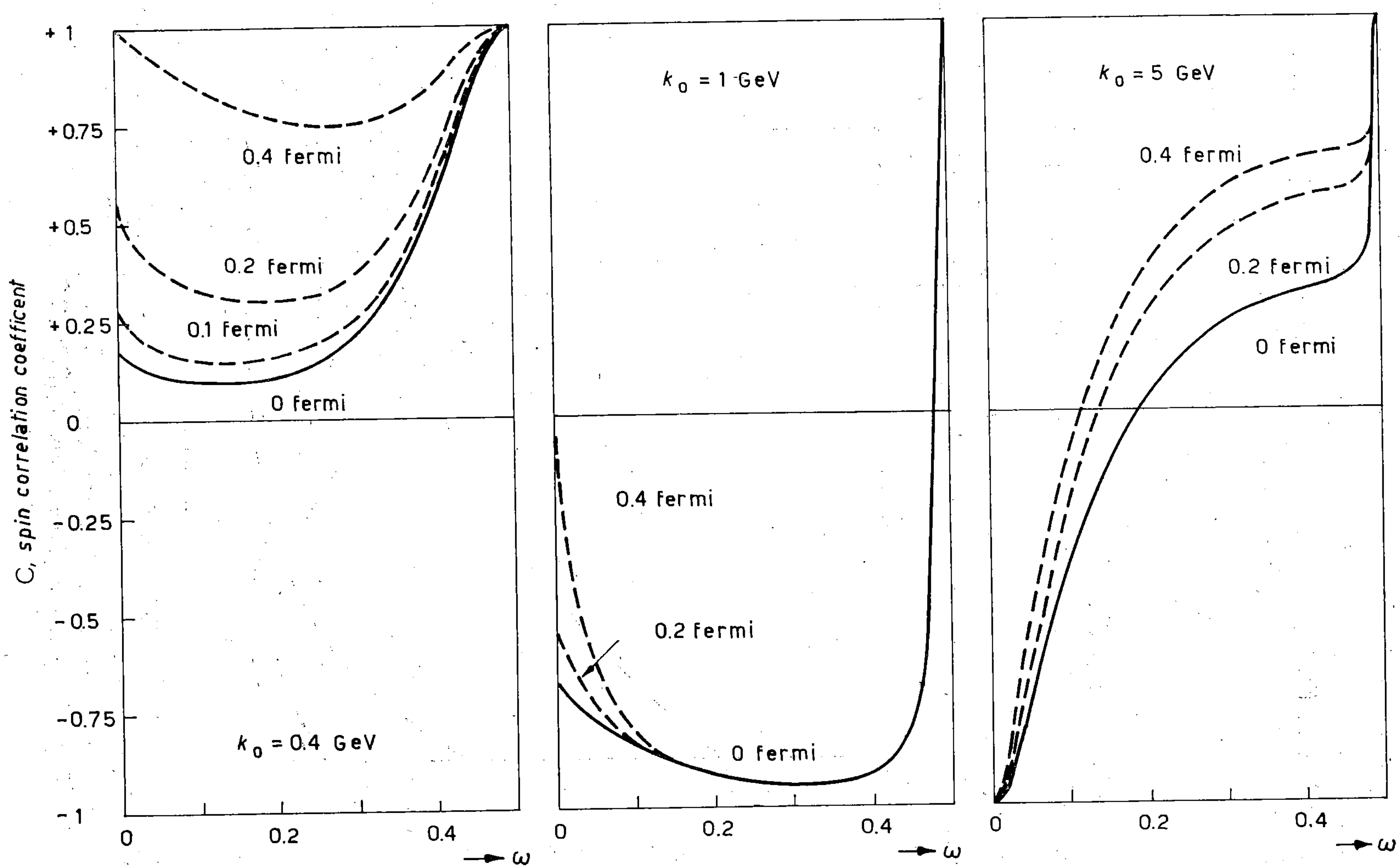

Fig. 5. – Spin correlation coefficient C for the coplanar longitudinal case with $\varphi_1=\varphi_2=10°$, as in Fig. 2, for different values of the cut-off parameter Λ.

fication of the muon-photon vertex or the muon and photon propagators. In (8) the photon propagator has divided out, therefore a modification of the photon propagator does not influence the spin-correlation coefficient, as long as we use lowest order perturbation theory. In order to estimate the effect of a modification of the muon propagator we follow DRELL [4] introducing a cut-off Λ by substituting in (4) and (8)

$$\frac{1}{2(qk)} \to \frac{1}{2(qk)} - \frac{1}{2(qk) - \Lambda^2}, \tag{9}$$

(and analogously for $1/(pk)$).

The numerical results for the coplanar symmetric case are given in Fig. 4 and 5.

For the transverse case and $\varphi_1 = \varphi_2 = 10°$ we find a significant cut-off-dependence only for very low and for high photon energies. For photon energies

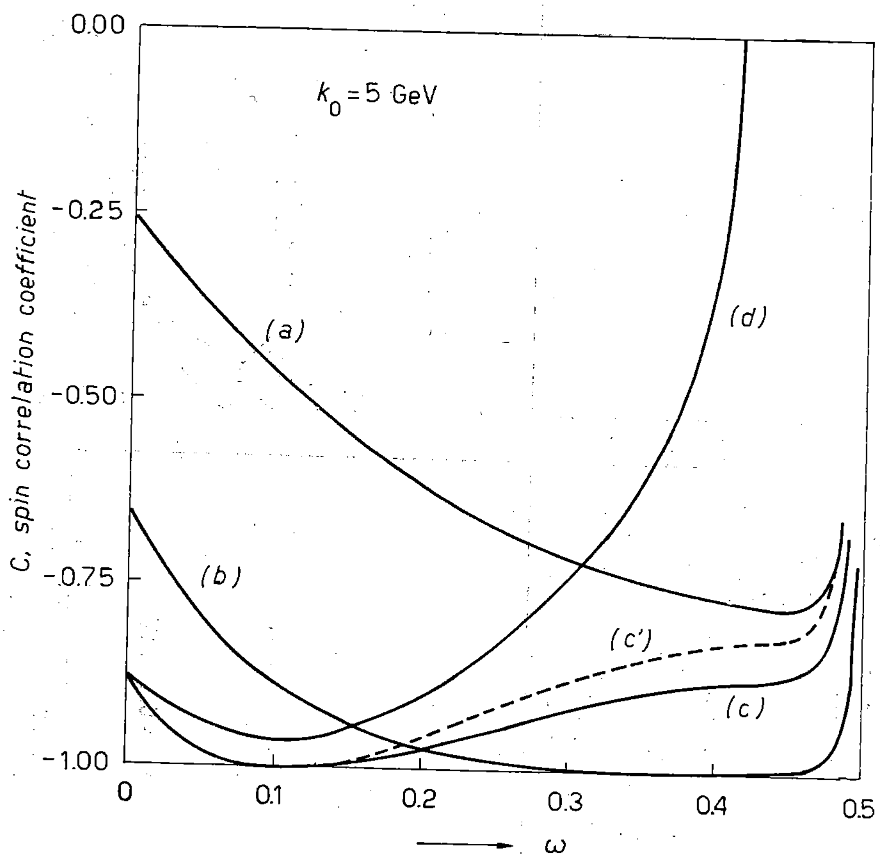

Fig. 6. – Spin correlation coefficient C for longitudinal spins at $k_0 = 5$ GeV, always $\varphi_2 = \varphi_1$: a) $\varphi_1 = 1.5°$; b) $\varphi_1 = 2.5°$; c) $\varphi_1 = 3.5°$; c') $\varphi_1 = 3.5°$ as curve c) but with a cut-off $\Lambda^{-1} = 0.2$ fermi; curves a) to c') all for $\boldsymbol{k}, \boldsymbol{p}, \boldsymbol{q}$ coplanar. Curve d): $\varphi_1 = \varphi_2 = 3.55°$, $\boldsymbol{k}, \boldsymbol{p}$ and $\boldsymbol{q}$ not coplanar, but $\sphericalangle(\hat{k}\text{-}\hat{p}), (\hat{q}\text{-}\hat{k}) = 19.5°$.

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[4] S. D. DRELL: *Ann. of Phys.*, **4**, 75 (1958).

between 0.5 and 2 GeV a cut-off at 0.2 fermi produces only an absolute change of less than 8% in the correlation coefficient.

For longitudinal spins there is a somewhat stronger cut–off-dependence as shown by Fig. 5. In this case too, high photon energies and photon energies near threshold are more favorable.

An experimental investigation of the muon-spin correlation would be very interesting, because here we can separate the effect of a modification of the photon propagator from an eventual muon anomaly. One need not worry about the nuclear form factor. Only relative, though difficult measurements are required. Of course, much effort is needed to reach the precision of the $g-2$ experiments ([5]).

At high energies a difficulty arises, because one has to use counters which cover a solid angle over which the spin correlation already varies strongly. An example for this case is given in Fig. 6, where the spin correlation at $k_0 = 5$ GeV is plotted for four different geometrical situations. The values of c for $\omega < 0.05$ are extrapolated by the eye. Cases b), c), and d) are all accepted by two 0.6 m sterad counters placed at 3° with respect to the photon beam. For a comparison of (8) with experiments at high energies one should therefore integrate (8) over the solid angle accepted by the counters.

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([5]) G. CHARPAK, F. J. M. FARLEY, R. L. GARWIN, T. MULLER, J. C. SENS and A. ZICHICHI: CERN 2878/NP.

RIASSUNTO

Si calcola, nella approssimazione di Born, la correlazione degli spin di muoni prodotti in coppie da fotoni non polarizzati. Si trova che il coefficiente di correlazione longitudinale è sensibile ad un cut-off nel propagatore del muone di 0.1 fermi.